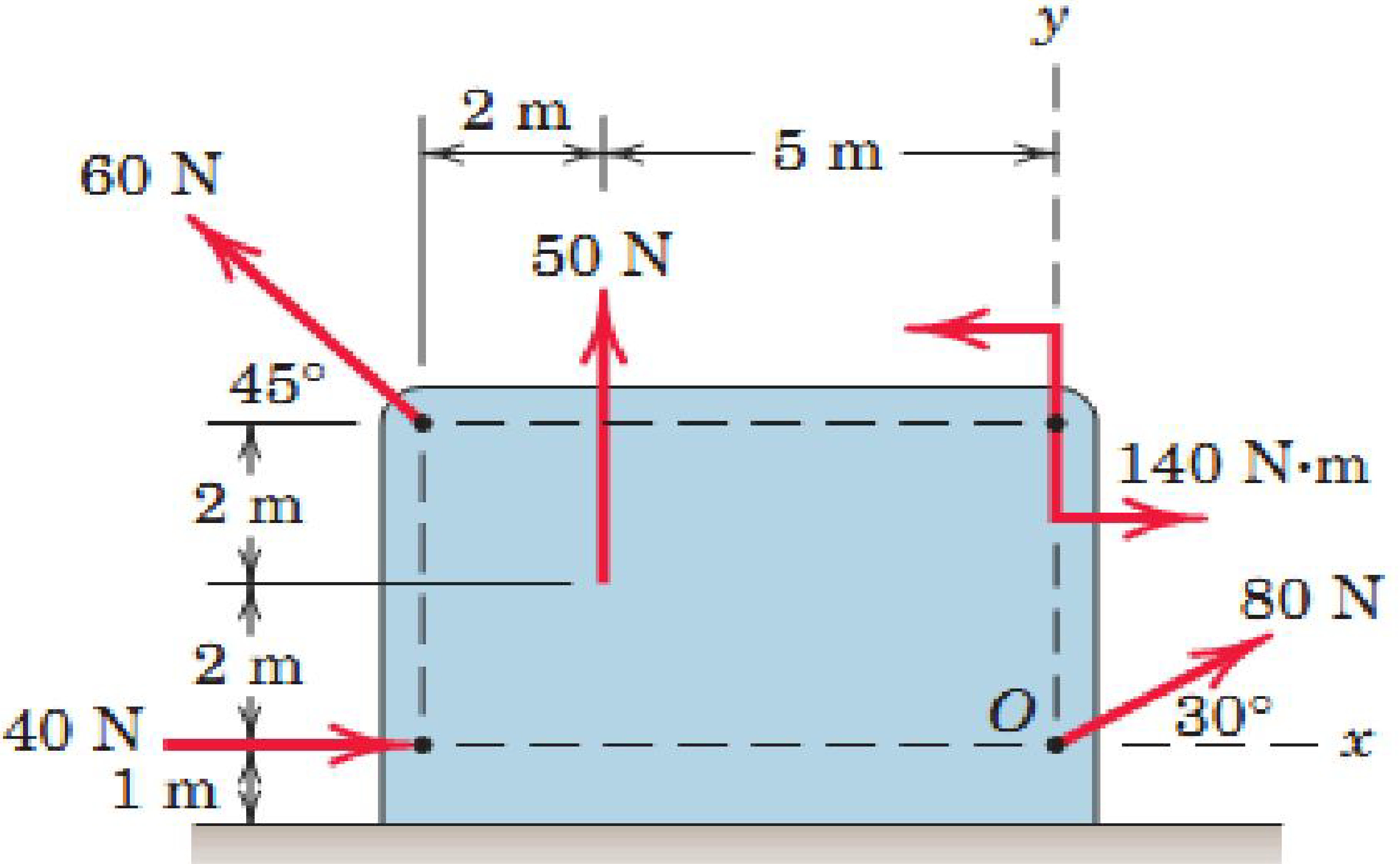
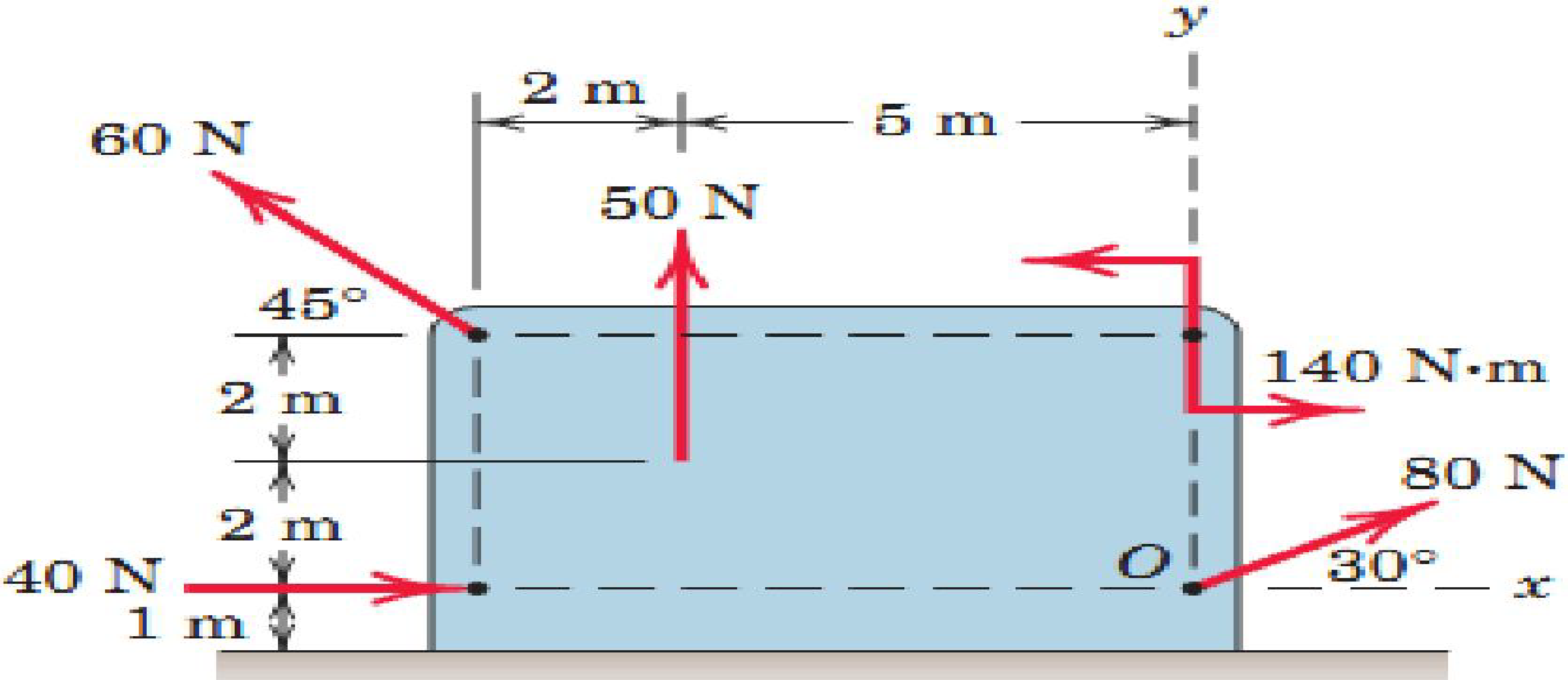
Reduce the system to a force and couple at o. then find the intersection of a single resultant force with x axis and y axis.





Rx=ΣFx=40+80 cos 30-60 cos 45=66.9 N⟶

Ry=ΣFy=50+80 sin 30+60 cos 45=132.4 N↑

$$R = \sqrt{Rx^2 + Ry^2} == \sqrt{66.9^2 + 132.4^2} = 148.3\ N$$

$$\tan \Theta = \frac{R_y}{R_x} = \frac{132.4}{66.9}$$

↺Mo=Σ(Fd)= 140-50(5)+60 cos 45*(4) -60 sin 45*(7)= -237 N.m↻

To find the location of the single resultant with x and y axis

1- put F_R in a vector form

$\vec{R} = \overrightarrow{R_X i} + \overrightarrow{R_Y\ j} = 66.9\ i + 132.4$ j

2- put $(M_R)_0$ in a vector form

$\overrightarrow{(M)_0} = -237$ k

- to find point m= (x i + y j) (new point)

$\overrightarrow{(M_R)_0}$ = $\vec{r}\ X \vec{R}$ $= \overrightarrow{om}\ X \vec{R}$

= (x i + y j) X ($66.9\ i +$ 132.4 j) = $-\ 237$ k

x*132.4 - y*66.9 =-237 (1)

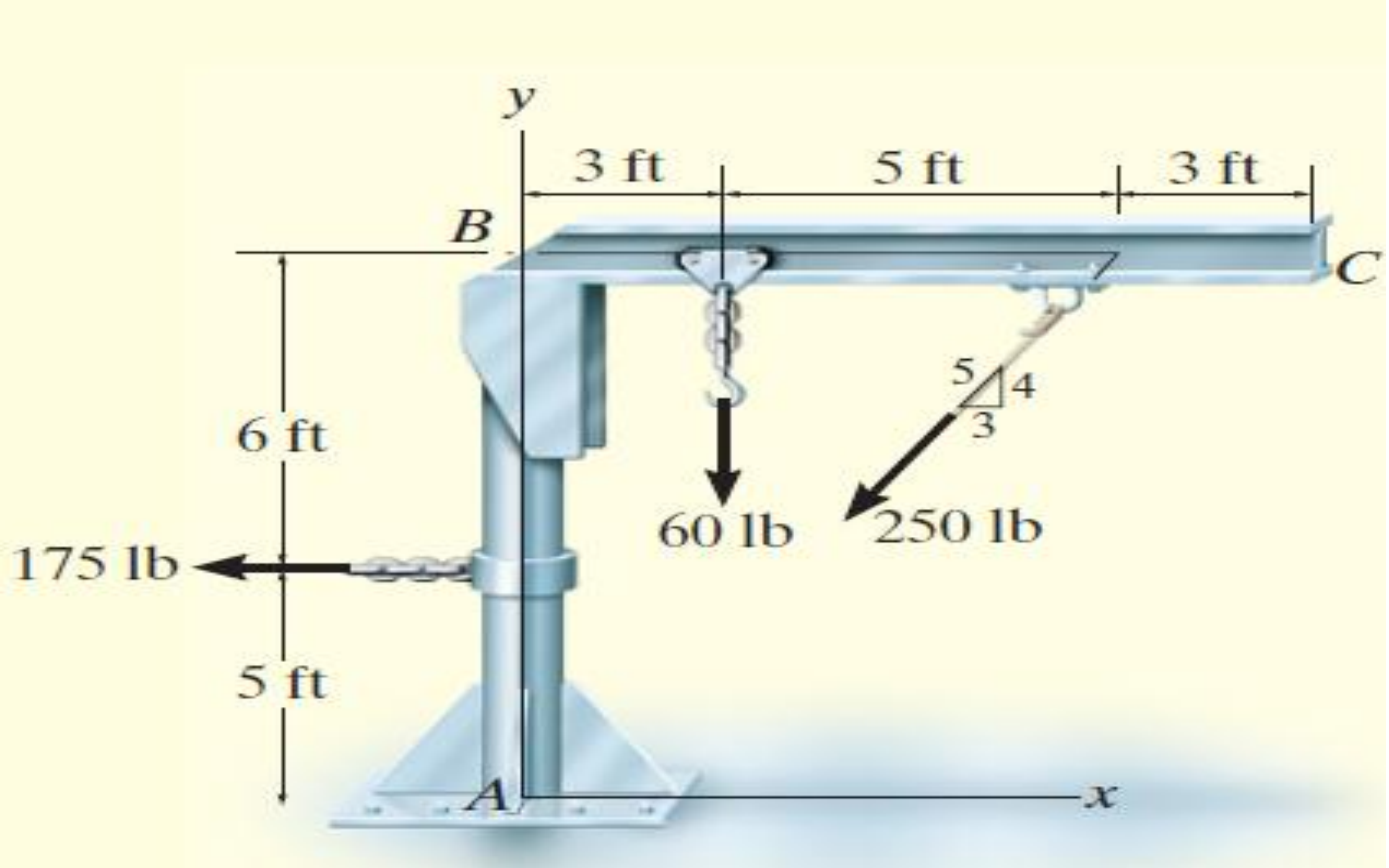
- Intersection with x axis at y=0 from (1)

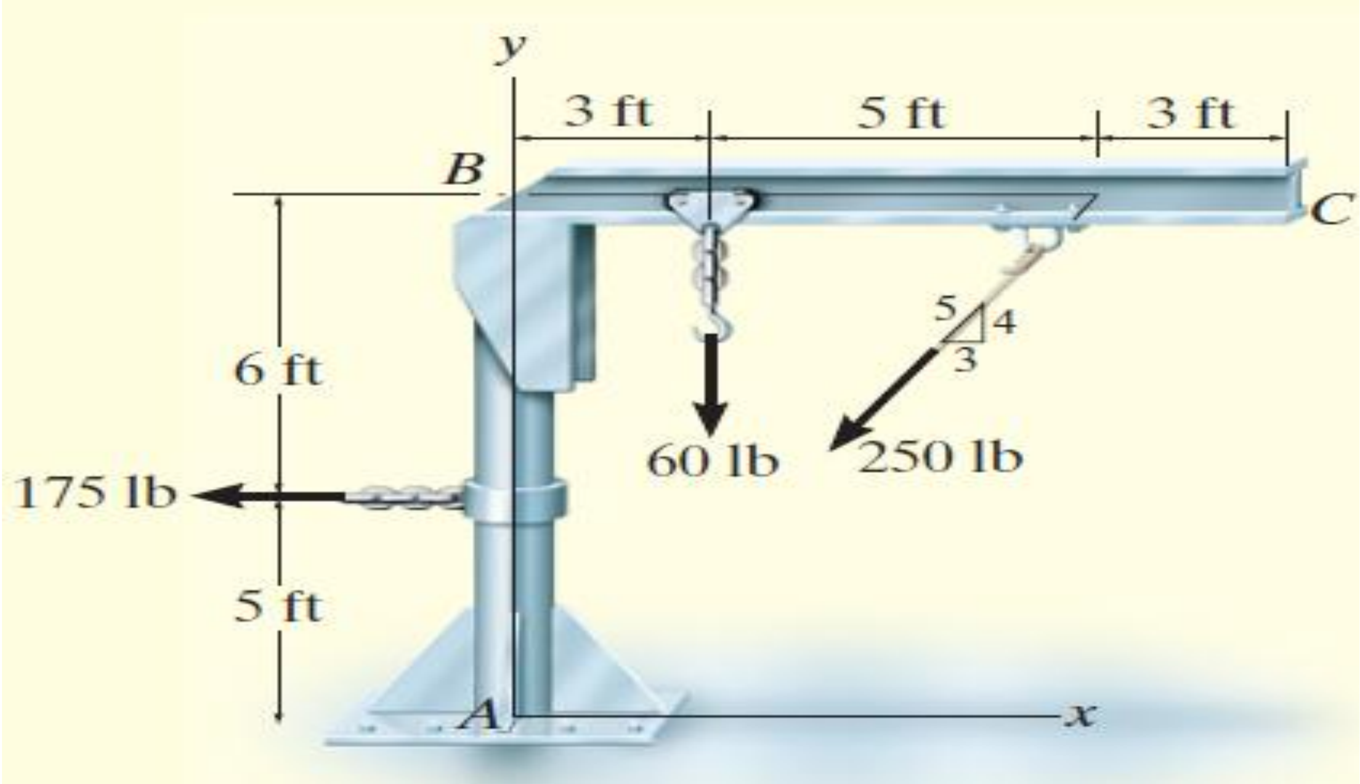
x = -237 / 132.4 = -1.79 (point m =(-1.79 ,0))

Intersection with y axis at x=0 from (1)

y= -237 /-66.9 = 3.54 (point m =(0, 3.54))

Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column *AB* and *BC*.





$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x;\ F_{R_x} = -250\text{ lb}\left(\frac{3}{5}\right) - 175\text{ lb} = -325\text{ lb} = 325\text{ lb} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y;\ F_{R_y} = -250\text{ lb}\left(\frac{4}{5}\right) - 60\text{ lb} = -260\text{ lb} = 260\text{ lb}\downarrow$$

$$F_R = \sqrt{(325)^2 + (260)^2} = 416\text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{260}{325}\right) = 38.7^\circ\ \theta$$

$$\circlearrowright + M_{R_A} = \Sigma M_A = 175\text{ lb}\,(5\text{ ft}) - 60\text{ lb}\,(3\text{ ft}) + 250\text{ lb}\left(\frac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\frac{4}{5}\right)(8\text{ ft})$$

= 745 lb .ft

To reduce the system to single resultant force

$\overrightarrow{M_{RA}} = \vec{r}\; x\; \overrightarrow{F_R} = \overrightarrow{Am}\; x \overrightarrow{F_R}$ *where* $m = (x\, i + y\, j)$ *is the new point*

$\overrightarrow{M_{RA}}$ =745 k = ($x\, i + y\, j$) x (-325 i -260 j) = (325 y-260 x) k

(325 y-260 x) = 745 (1)

(325 y-260 x) = 745 (1)

Intersection with AB at x=0

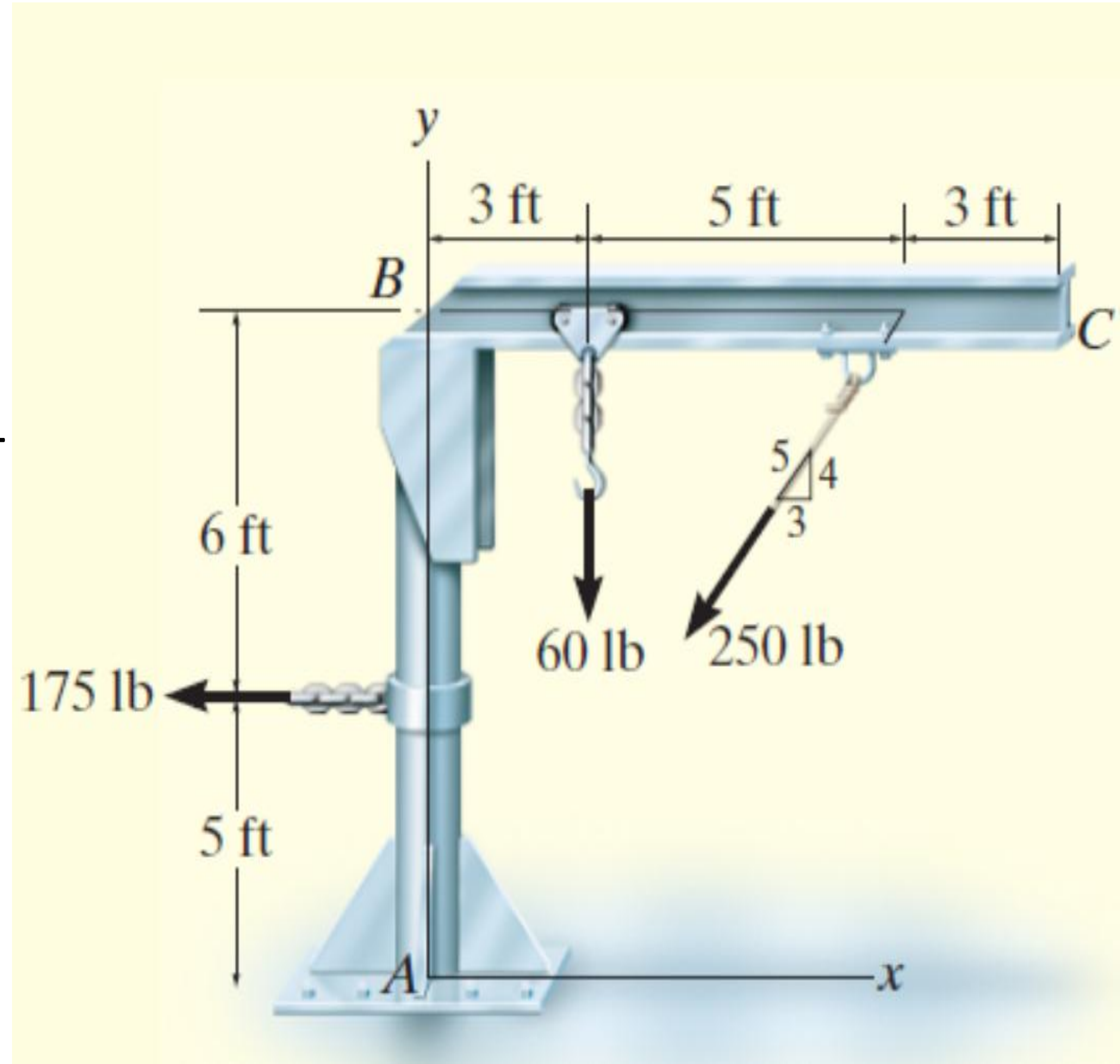
325 y = 745

y= 2.29 ft

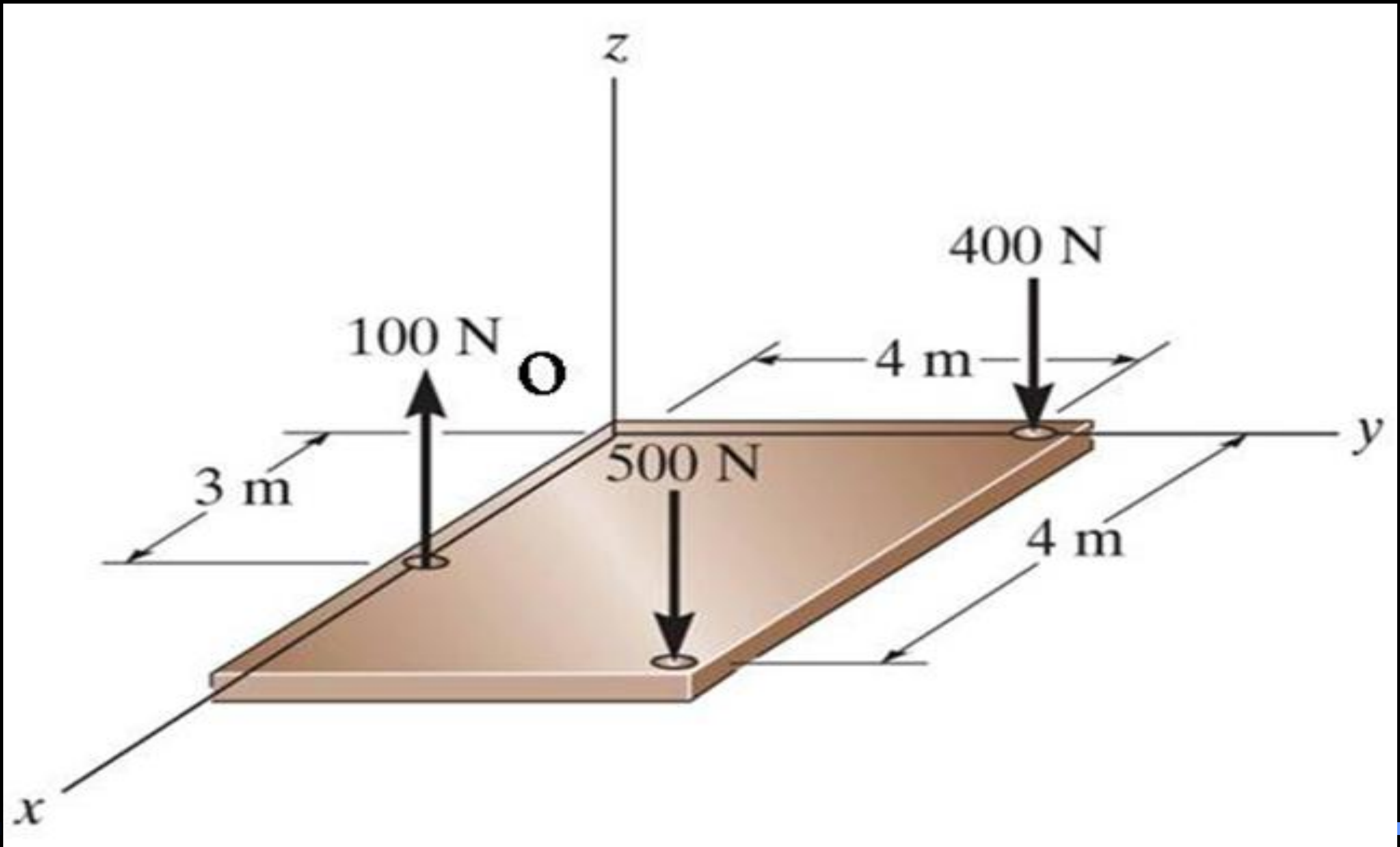
Intersection with BC at y=11

(325 *11 -260 x) = 745

x = 10.9 ft



Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x, y) of the single equivalent resultant force.



$\overrightarrow{F_1} = 100\,k$ & $\overrightarrow{F_2} = -500\,k$ & $\overrightarrow{F_3} = -400\,k$

$\overrightarrow{r_1} = 3\,i$ & $\overrightarrow{r_2} = (4\,i + 4\,j)$ & $\overrightarrow{r_3} = (4\,j)$

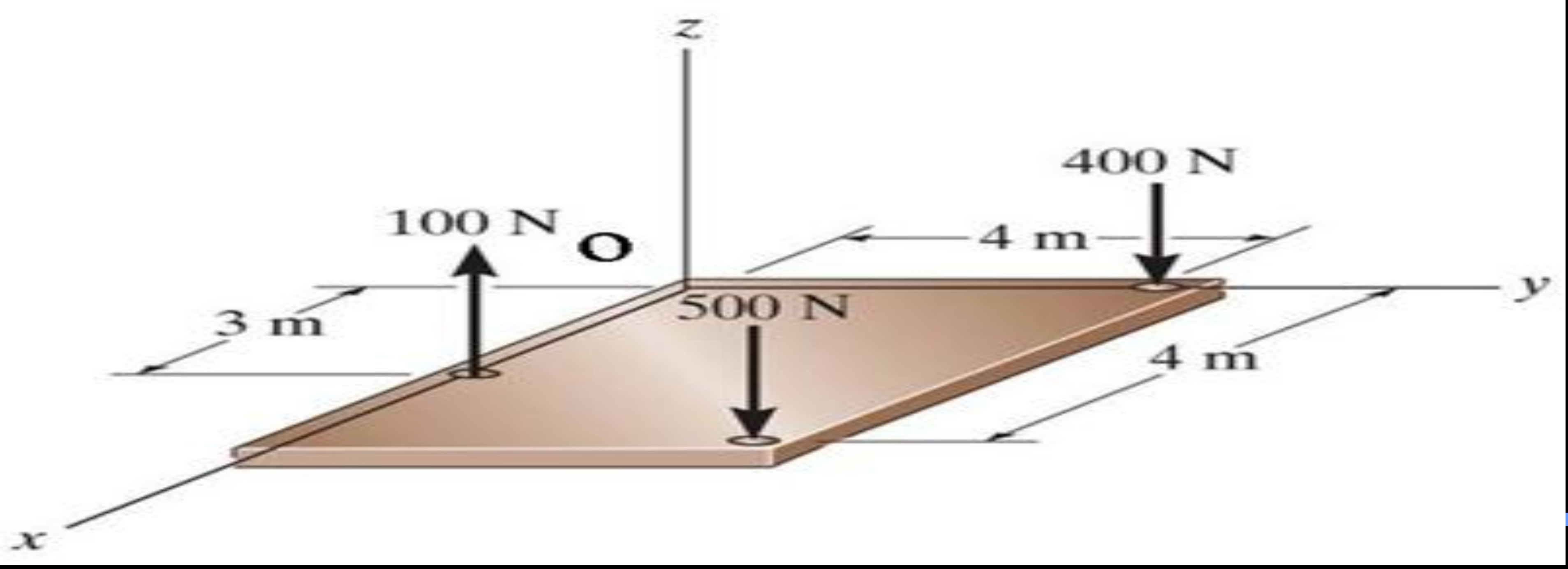
$F_{RO} = \{100\,k - 500\,k - 400\,k\} = -800\,k$ N

$M_{RO} = (3\,i) \times (100\,k) + (4\,i + 4\,j) \times (-500\,k)$

$+ (4\,j) \times (-400\,k)$

$= \{-300\,j + 2000\,j - 2000\,i - 1600\,i\}$

$= \{-3600\,i + 1700\,j\}$ N·m



$\boldsymbol{F_{RO}} = -\mathbf{800}\ \boldsymbol{k}\,\mathrm{N}$

$\boldsymbol{M_{RO}} = \{-\mathbf{3600}\ \boldsymbol{i} + \mathbf{1700}\ \boldsymbol{j}\}$

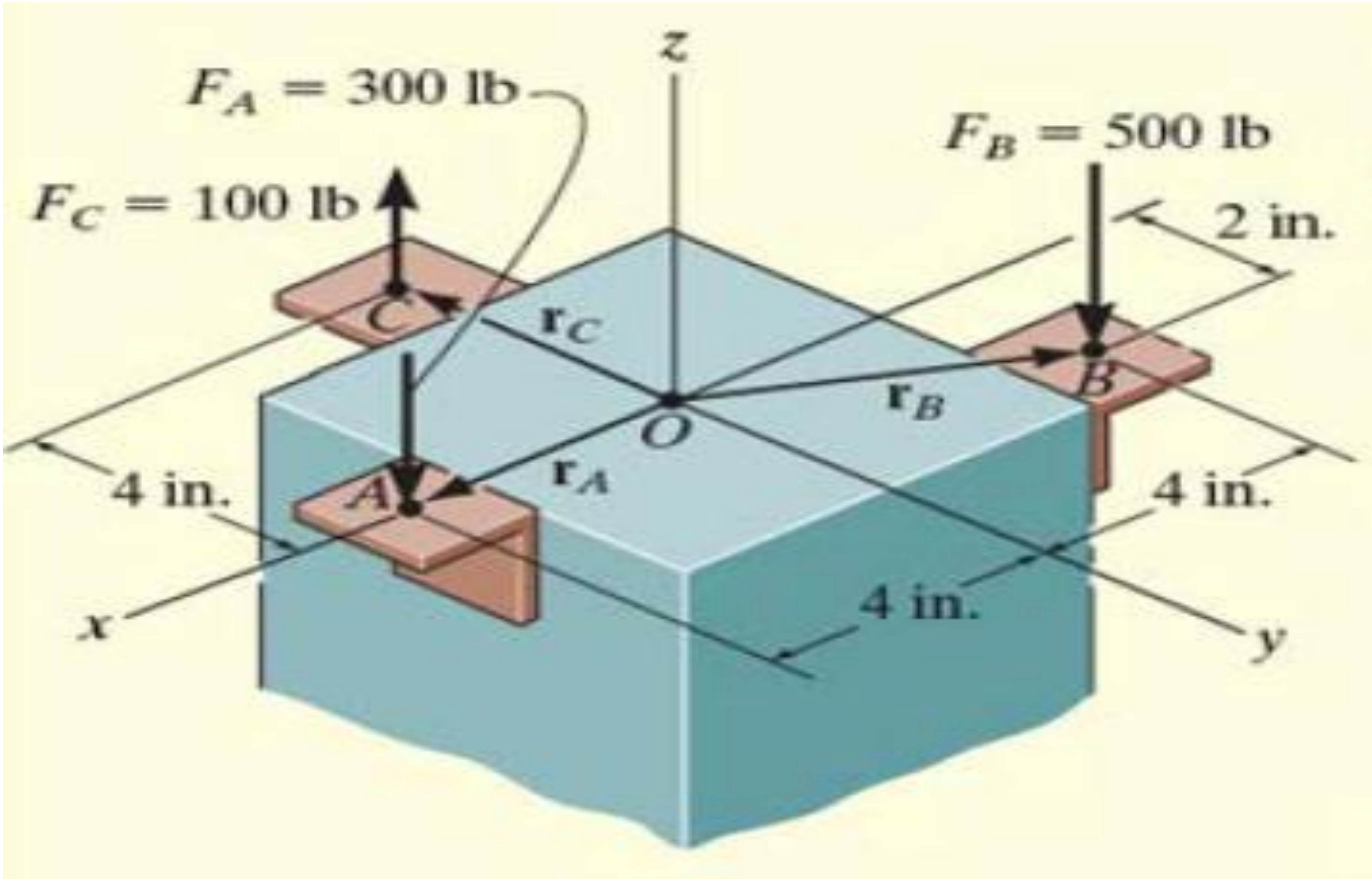
to find point p= (x i + y j + 0 k) (new point) (same like moment) & o = (0,0,0)

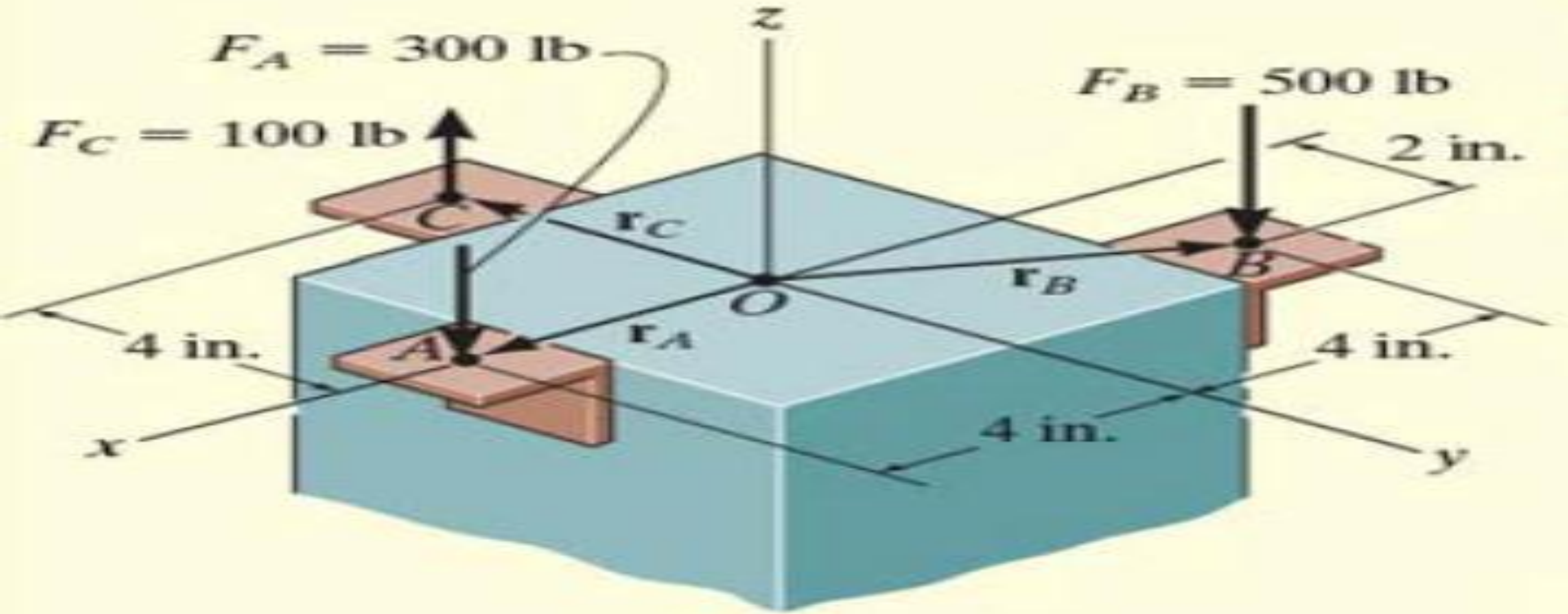
$\overrightarrow{(M_R)_0} = \vec{r}\ X\overrightarrow{F_R} = \overrightarrow{op}\ X\overrightarrow{F_R}$

- = (x i + y j) X (F_R k) $= M_{RX}\ i + M_{RY}\ j$
- = (x i + y j) X (-800 k) $= -3600\ i + 1700\ j$
- y$*$ (F_R) $= M_{RX}$

  y= -3600/-800 =4.5 m
- –x $*$(F_R) $= M_{RY}$
- -x= 1700 / -800
- **x= 2.13 m**
- **P=(2.13 , 4.5, 0)**

Replace the system by a force and find its point of application





$\mathbf{F}_R = \Sigma\mathbf{F};$ $\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-300\mathbf{k}\}\text{ lb} + \{-500\mathbf{k}\}\text{ lb} + \{100\mathbf{k}\}\text{ lb}$

$= \{-700\mathbf{k}\}\text{ lb}$

$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;$ $= (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) + (\mathbf{r}_C \times \mathbf{F}_C)$

$= [(4\mathbf{i}) \times (-300\mathbf{k})] + [(-4\mathbf{i} + 2\mathbf{j}) \times (-500\mathbf{k})] + [(-4\mathbf{j}) \times (100\mathbf{k})]$

$= -1200(\mathbf{i} \times \mathbf{k}) + 2000(\mathbf{i} \times \mathbf{k}) - 1000(\mathbf{j} \times \mathbf{k}) - 400(\mathbf{j} \times \mathbf{k})$ $= 1200\mathbf{j} - 2000\mathbf{j} - 1000\mathbf{i} - 400\mathbf{i}$

- **To find the point of application of the resultant force**

$$\mathbf{r}_P \times \mathbf{F}_R = 1200\mathbf{j} - 2000\mathbf{j} - 1000\mathbf{i} - 400\mathbf{i}$$

$$= \overrightarrow{op}\, X \overrightarrow{F_R} = -1400\ i - 800\ j$$

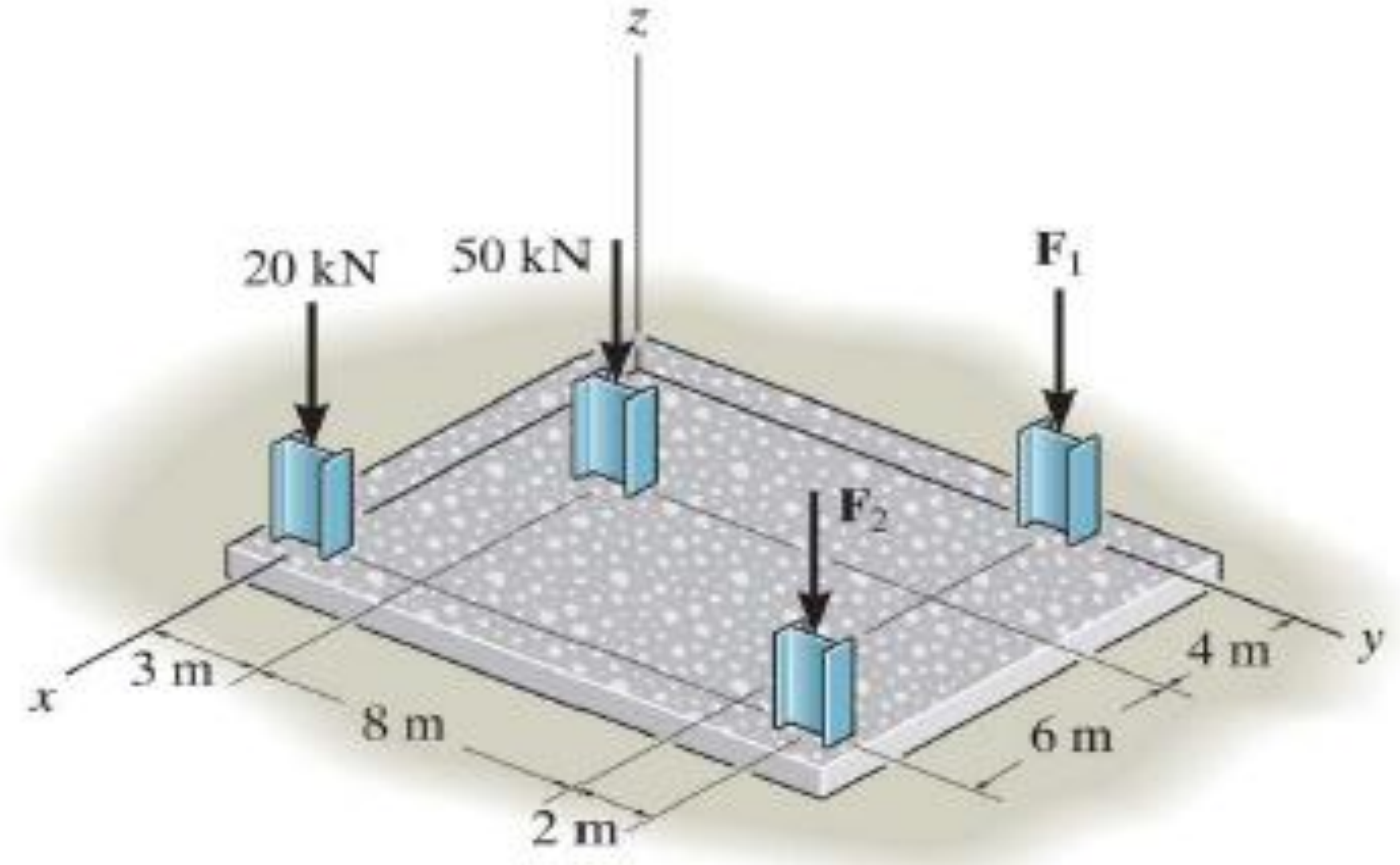
$$(x\mathbf{i} + y\mathbf{j}) \times (-700\mathbf{k}) = -1400\ i - 800\ j$$

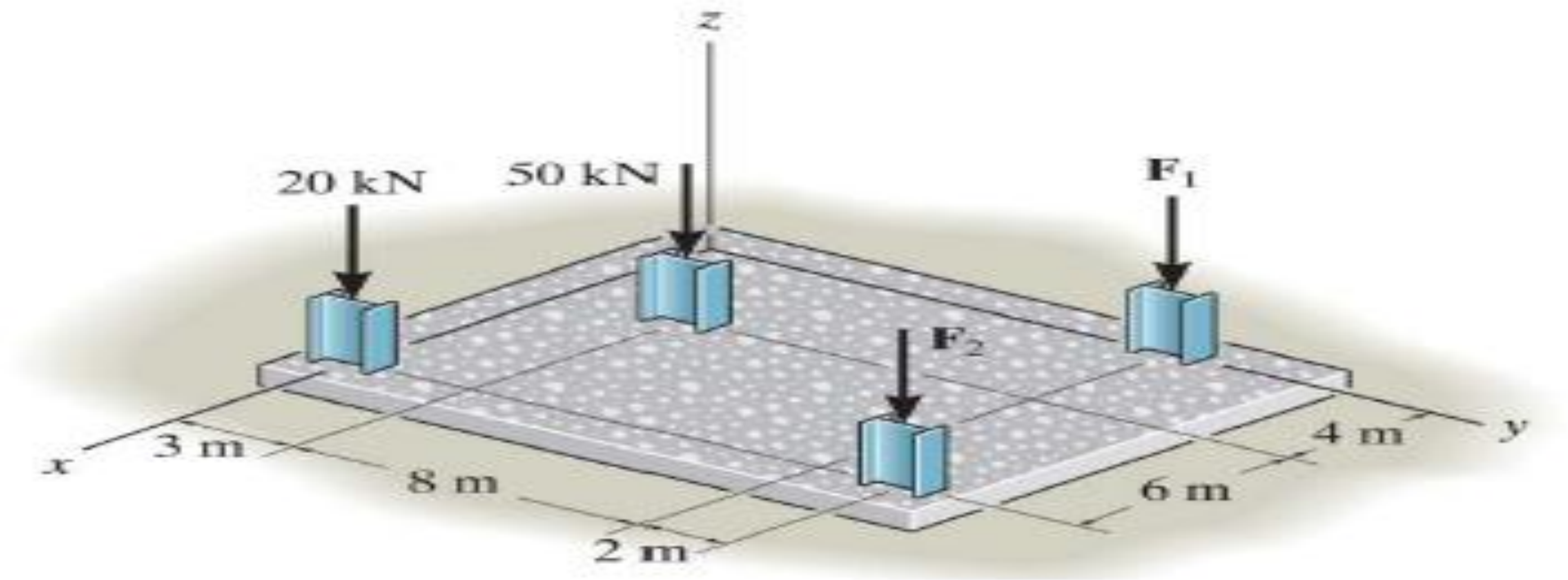
$$700x\mathbf{j} - 700y\mathbf{i} = -1400\ i - 800\ j$$

Equating the **i** and **j** components,

$$-700y = -1400$$
$$y = 2 \text{ in.}$$
$$700x = -800$$
$$x = -1.14 \text{ in.}$$

Given: F1 and F2 = 0. Find: The equivalent resultant force and couple moment at the origin *O*. Also find the location (*x*,*y*) of the single equivalent resultant force.





$F_1 = -20\ k$ & $F_2 = -50\ k$

$r_1 = 10\ i$ & $r_2 = 4i + \ 3\ j$

$F_{RO} = -20\ k - 50\ k$= -70 k KN

$M_{RO} = (10\ \boldsymbol{i}) \times (-20\ \boldsymbol{k}) + (4\ \boldsymbol{i} + 3\ \boldsymbol{j}) \times (-50\ \boldsymbol{k}) = \{200\ \boldsymbol{j} + 200\ \boldsymbol{j} - 150\ \boldsymbol{i}\}$ kN·m

$= \{-150\ \boldsymbol{i} + 400\ \boldsymbol{j}\}$ **kN·m**

F_{RO} =-70 k KN

$M_{RO} = -150\, i + 400\, j$

to find point p= (x i + y j + 0 k) (new point)

(same like moment) & o = (0,0,0)

$\overrightarrow{(M_R)_0}$ = $\vec{r}\ X \overrightarrow{F_R}$ $= \overrightarrow{op}\ X \overrightarrow{F_R}$

- = (x i + y j) X (F_R k) =$M_{RX}\ i +\ M_{RY}\ j$
- = (x i + y j) X ($-$ 70 k) = $-$ 150 i + 400 j
- y$*$ (F_R) =M_{RX}

  y= -150/-70 =2.14 m
- $-$x $*$(F_R) = M_{RY}
- -x= 400 / -70
- **x= 5.71 m**
- **P=(5.71 , 2.14, 0)**

Wrench reduction

1- we have $\overrightarrow{F_R}$ and $\overrightarrow{M_R}$ as a force and couple at a point

For a reduction to a wrench

2- **parallel moment** = $M_\parallel = \dfrac{\overrightarrow{F_R} \cdot \overrightarrow{M_R}}{\left|\overrightarrow{F_R}\right|}$ N.m

$\overrightarrow{M_\parallel} = \dfrac{\overrightarrow{F_R} \cdot \overrightarrow{M_R}}{\left|\overrightarrow{F_R}\right|^2} * \overrightarrow{F_R}$ N.m

Orthogonal moment = $\overrightarrow{M_\perp} = \overrightarrow{M_R} - \overrightarrow{M_\parallel}$ N.m

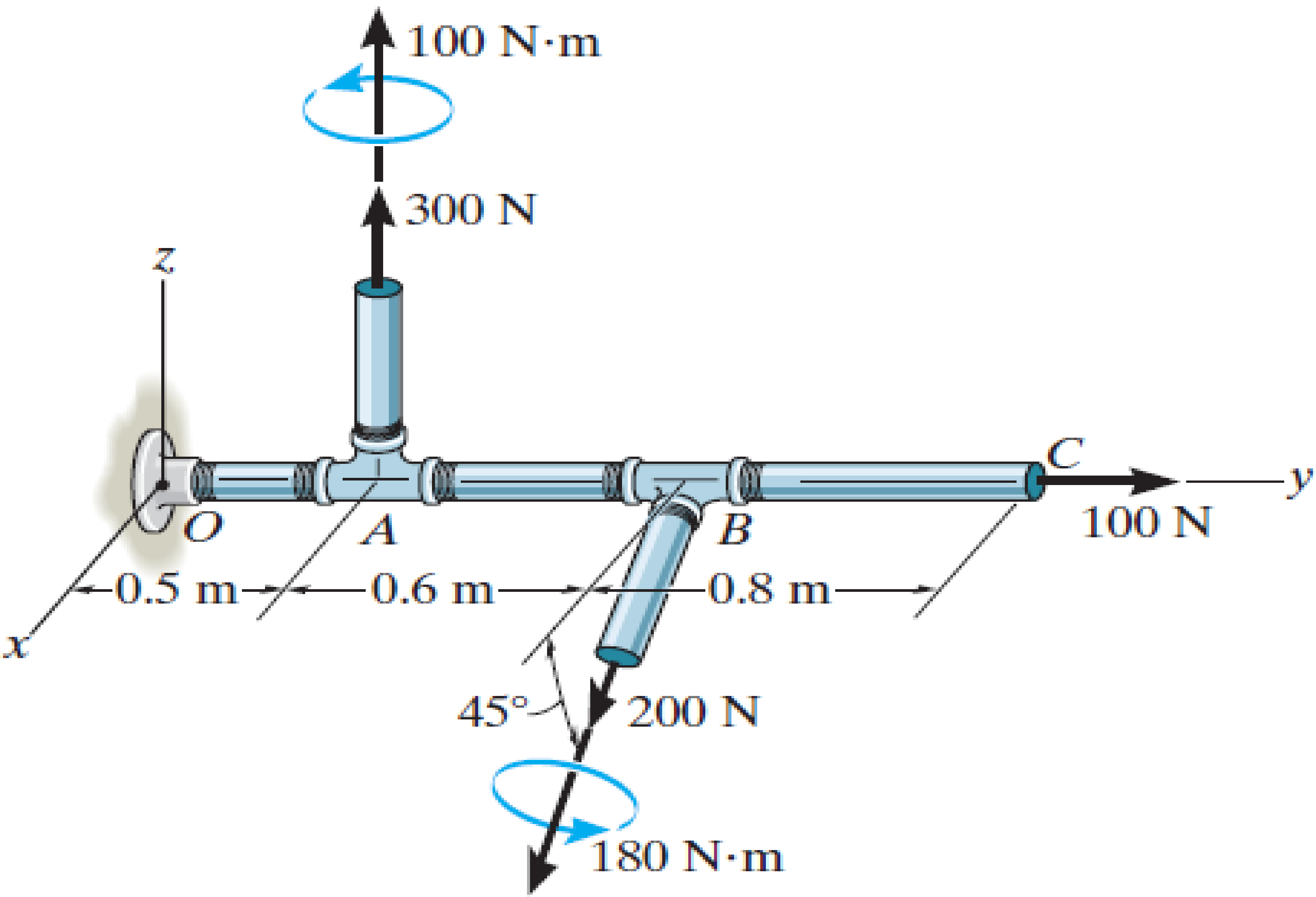
3- **Pitch** = P = $\dfrac{M_\parallel}{\left|\overrightarrow{F_R}\right|} = \dfrac{\overrightarrow{F_R} \cdot \overrightarrow{M_R}}{\left|\overrightarrow{F_R}\right|^2}$ m

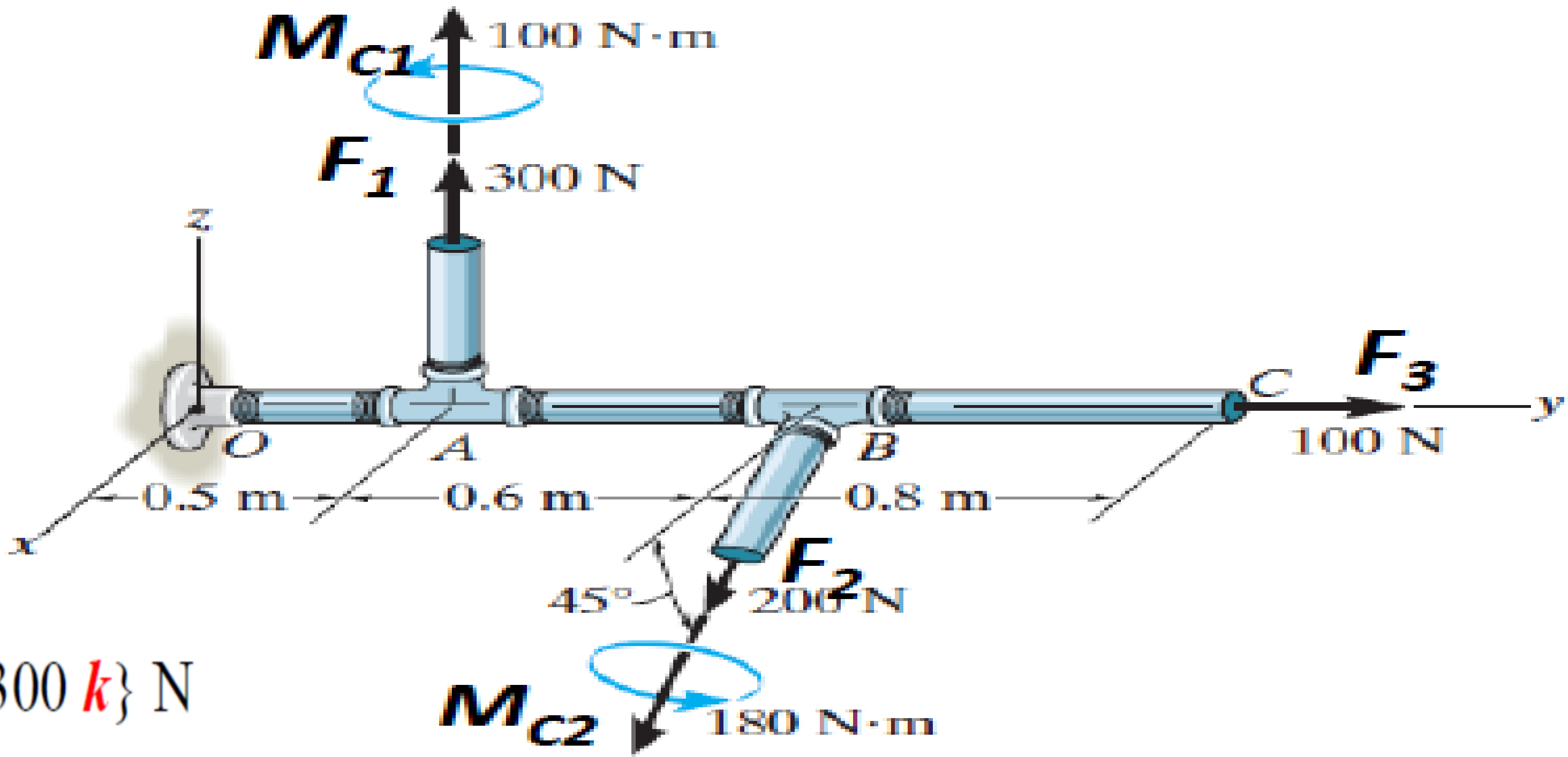
4- **strength** = $\left|\overrightarrow{F_R}\right|$ N

5- **Central axis=** $\vec{\boldsymbol{r}}$ = $\dfrac{\overrightarrow{F_R} \times \overrightarrow{M_R}}{\left|\overrightarrow{F_R}\right|^2} + \lambda\, \overrightarrow{F_R}$ m

(λ constant parameter)

Replace the system by a wrench(force and couple at o) and find the intersection of the wrench with x-y plane.





$F_1 = \{300\, \mathbf{k}\}$ N

$F_2 = 200\{\cos 45°\, \mathbf{i} - \sin 45°\, \mathbf{k}\}$ N $= \{141.4\, \mathbf{i} - 141.4\, \mathbf{k}\}$ N

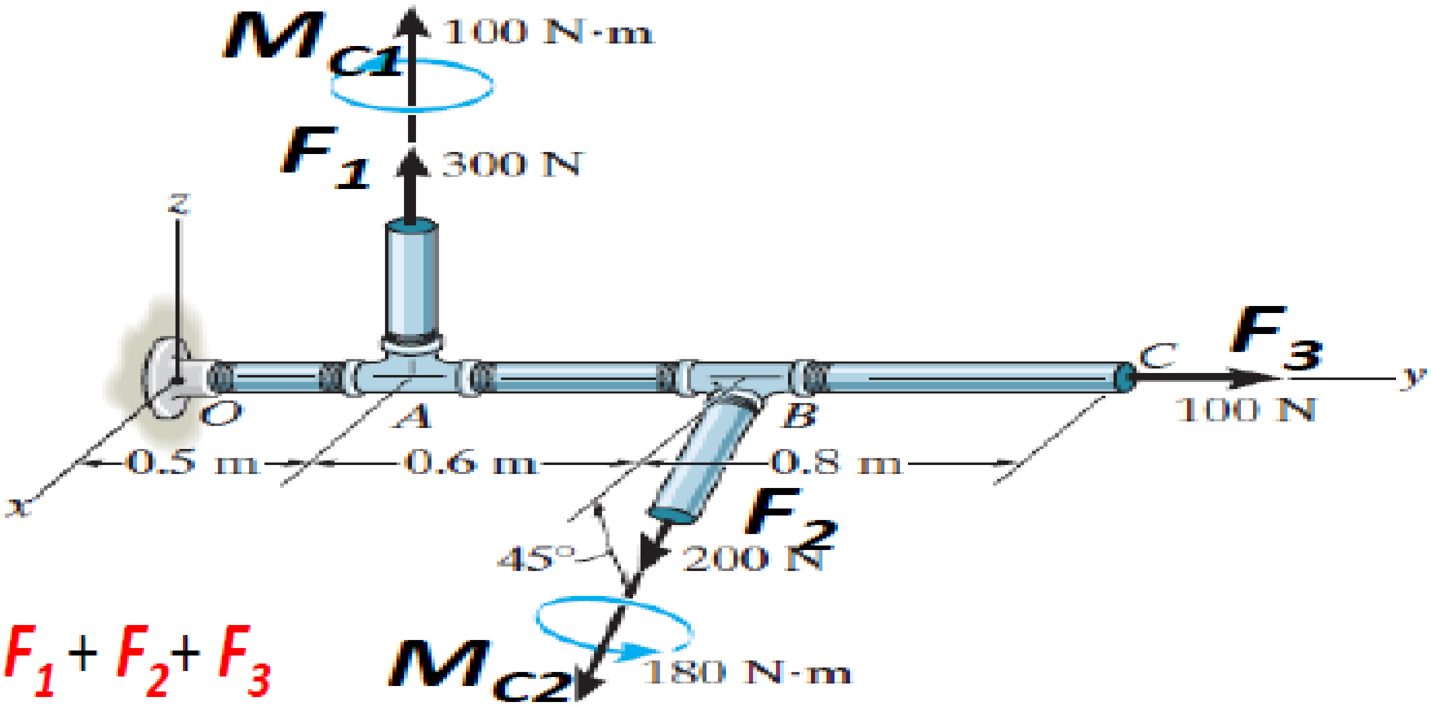
$F_3 = \{100\, \mathbf{j}\}$ N

$M_{C1} = \{100\, \mathbf{k}\}$ N·m

$M_{C2} = 180\{\cos 45°\, \mathbf{i} - \sin 45°\, \mathbf{k}\}$ N·m $= \{127.3\, \mathbf{i} - 127.3\, \mathbf{k}\}$ N·m

$r_1 = \{0.5\, \mathbf{i}\}$ m, $r_2 = \{1.1\, \mathbf{i}\}$ m,

$r_3 = \{1.9\, \mathbf{i}\}$ m



$$\mathbf{F}_{RO} = \sum \mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= \{300\,\mathbf{k}\} + \{141.4\,\mathbf{i} - 141.4\,\mathbf{k}\} \;\; + \{100\,\mathbf{j}\}$$

$$\mathbf{F}_{RO} = \{\underline{141}\,\mathbf{i} + \underline{100}\,\mathbf{j} + \underline{159}\,\mathbf{k}\}\ \underline{\text{N}}$$

$$M_{RO} = \sum M_C + \sum (\, r_i \times F_i \,)$$

$$= \{100\,k\} + \{127.3\,i - 127.3k\} \;\; + \begin{vmatrix} i & j & k \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 1.1 & 0 \\ 141.4 & 0 & -141.4 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 1.9 & 0 \\ 0 & 100 & 0 \end{vmatrix}$$

$$\mathbf{M}_{RO} = \{122\,\mathbf{i} - 183\,\mathbf{k}\}\ \underline{\text{N·m}}$$

$\overrightarrow{F_{RO}}$ = (141,100,159) & $\overrightarrow{M_{RO}}$ =(122,0,-183)

For the wrench

1- **strength** $= \left|\overrightarrow{F_R}\right|$= $\sqrt{141^2 + 100^2 + 159^2}$= 234.86 N

2- **parallel moment** = $M_{\parallel} = \frac{\overrightarrow{F_R} \cdot \overrightarrow{M_R}}{\left|\overrightarrow{F_R}\right|}$

$= \frac{(141,100,159) \cdot (122,0,-183)}{234.86}$ = -50.65 N.m

$\overrightarrow{M_{\parallel}} = M_{\parallel} * \frac{\overrightarrow{F_R}}{\left|\overrightarrow{F_R}\right|} = -50.65 * \frac{(141,100,159)}{234.86}$= (-31,-22,-35) N.m

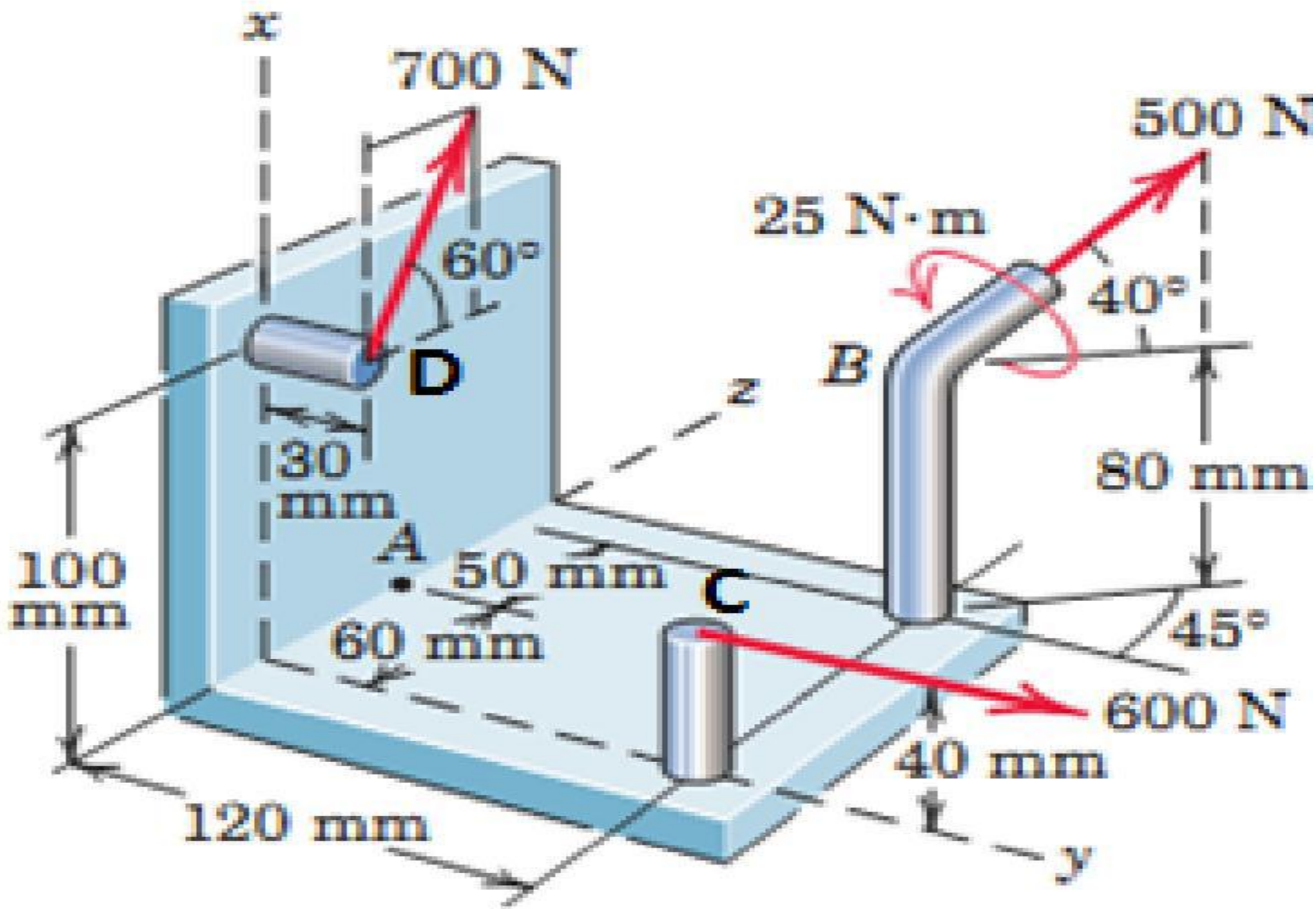
Orthogonal moment = $\overrightarrow{M_{\perp}} = \overrightarrow{M_R} - \overrightarrow{M_{\parallel}}$

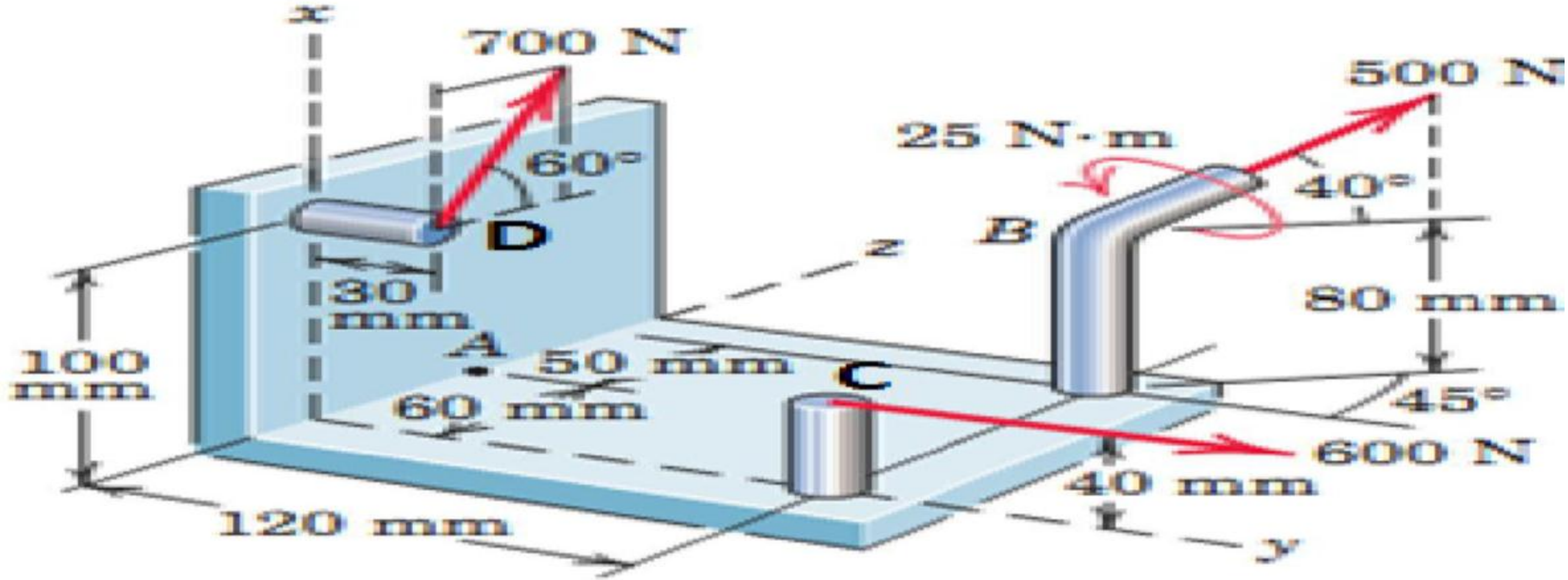
=(122,0,-183)- (-31,-22,-35) = (153,22,-148) N.m

3- **Pitch** = P = $\frac{M_{\parallel}}{\left|\overrightarrow{F_R}\right|} = \frac{50.65}{234.86}$= 0.22 m

- 4- **Central axis=** $\vec{r} = \frac{\overrightarrow{F_R} \times \overrightarrow{M_R}}{\left|\overrightarrow{F_R}\right|^2} + \lambda \overrightarrow{F_R}$ (λ constant parameter)
- $= \frac{(141,100,159)\ x\ (122,0,-183)}{234.86^2} + \lambda\ (141,100,159)$
- =(-0.33, 0.82,-0.22) + λ (141,100,159)
- ***The intersection with the x-y plane***
- The point at the plane x-y = (x1,y1,0)
- x1= -0.33 +141 λ (1)
- y1= 0.82 +100 λ (2)
- 0= -0.22 +159 λ (3)
- From (3) λ =0.22/159=1.4/1000
- From (1) x1= -0.133 & From (2) y1 =0.96
- The point of intersection =(-0.133,0.96,0)

Replace the system by a wrench(force and couple at A) and find the intersection of the wrench with x-y plane.





- $F_{500} = 500(sin40, cos40cos45, cos40sin45)$
- $F_{600} = 600j$
- $F_{700} = 700(sin60,0, cos60)$
- $r_{AB} = \ (0.08,0.12,0.05)$
- $r_{AC} = \ (0.04,0.12, -0.06)$
- $r_{AD} = \ (0.1,0.03, -0.06)$
- $\vec{R} = F_{500}$ $+F_{600}$$+F_{700}$ = (928,871,621)

$$\vec{M}_{500} = \vec{r}_{AB} \times \vec{F}_{500} =$$

$$(0.08-0)i + (0.12-0)j + (0.11-0.06)k \times 500(i\sin 40 + j\cos 40\cos 45 + k\cos 40\sin 45)$$

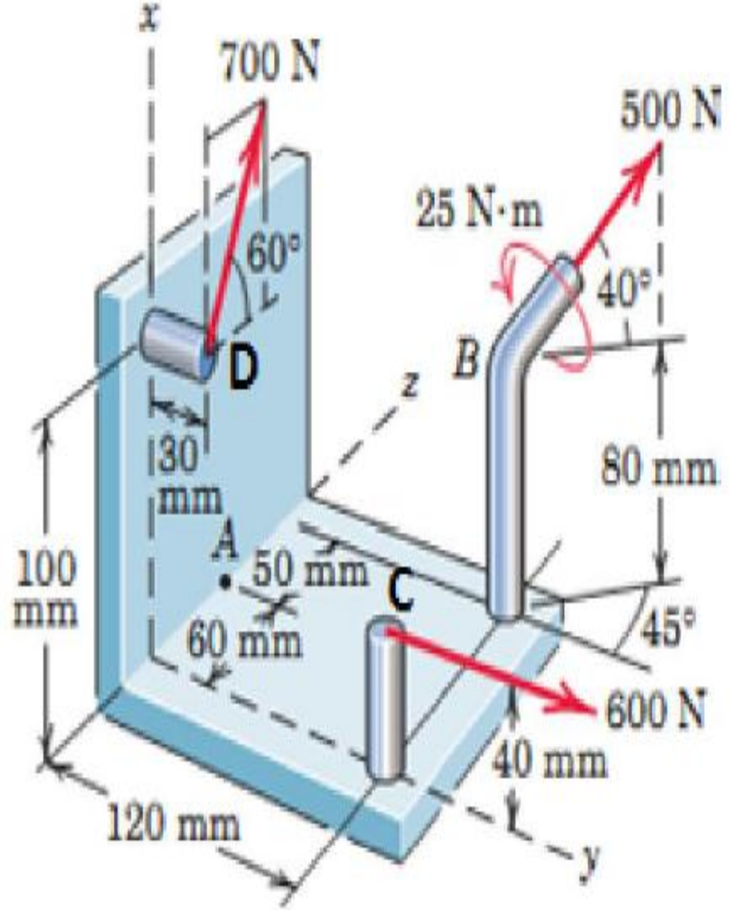
$$= \begin{vmatrix} i & j & k \\ 0.08 & 0.12 & 0.05 \\ \sin 40 & \cos 40\cos 45 & \cos 40\sin 45 \end{vmatrix}_{*500} = 18.96i - 5.59j - 16.9k \;\; N.m$$

$$\vec{M}_{600} = \vec{r}_{AC} \times \vec{F}_{600} = (0.04i + 0.12j - 0.06k) \times 600(0i + 1j + 0k)$$

$$= \begin{vmatrix} i & j & k \\ 0.04 & 0.12 & -0.06 \\ 0 & 600 & 0 \end{vmatrix}_{*600} = 36i + 24k \;\; N.m$$

$$\vec{M}_{700} = \vec{r}_{AD} \times \vec{F}_{700} = (0.1i + 0.03j - 0.06k) \times 700(\sin 60\, i + 0j + \cos 60\, k)$$

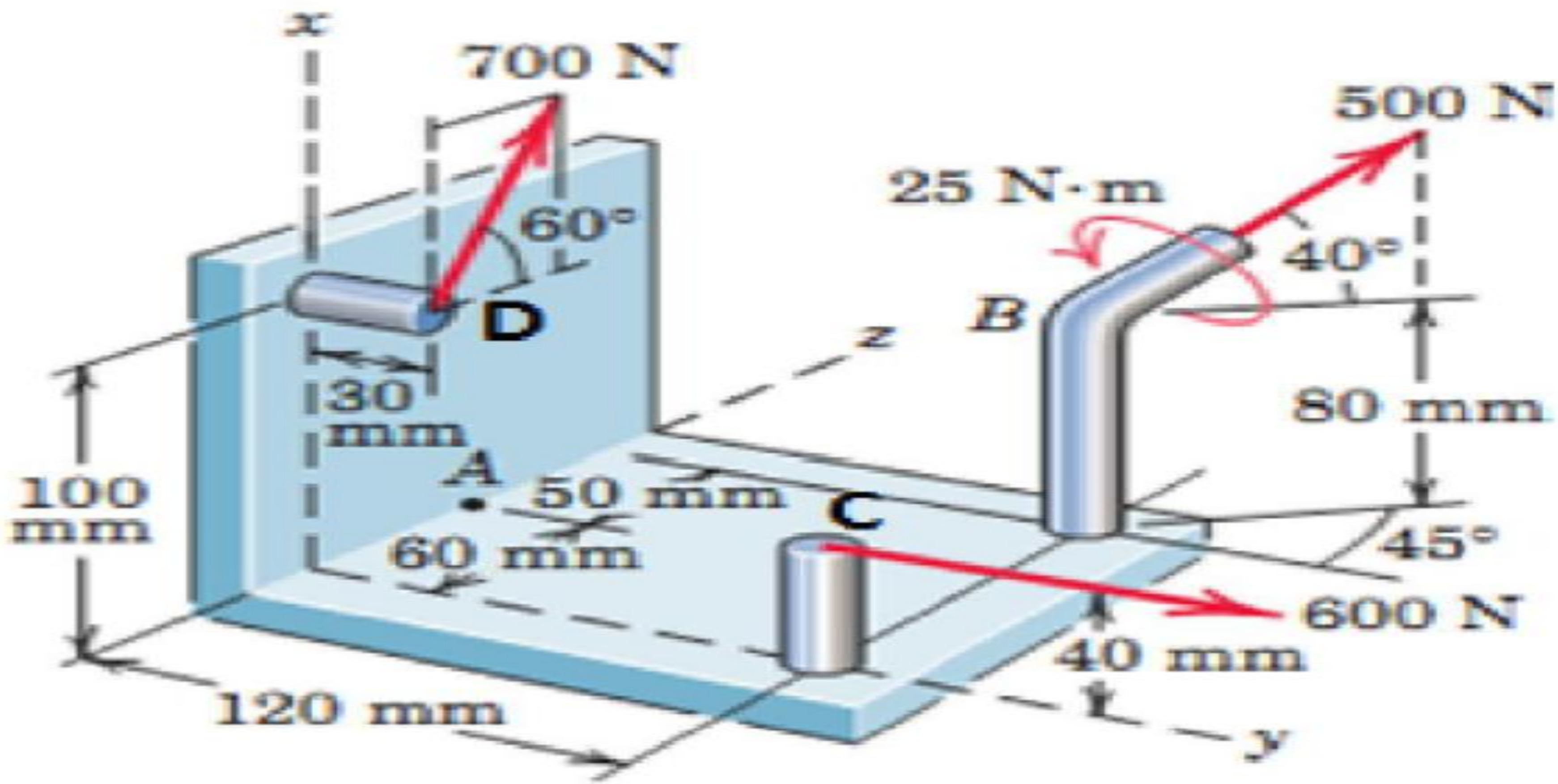
$$= \begin{vmatrix} i & j & k \\ 0.1 & 0.03 & -0.06 \\ \sin 60 & 0 & \cos 60 \end{vmatrix}_{*700} = 10.5i - 71.4j - 18.19k \;\; N.m$$



The couple of the given wrench

$$\vec{M}' = 25(-\sin 40\, i - \cos 40 \cos 45\, j - \cos 40 \sin 45\, k)$$
$$= -16.07i - 13.54j - 13.54k \ \ N.m$$

$$\vec{M}_A = \vec{M}_{500} + \vec{M}_{600} + \vec{M}_{700} + \vec{M}' \quad = 49.4i - 90.5j - 24.6k \ \ N.m$$



- $\vec{R}$= (928,871,621) & $M_A = (49.4, -90.5, -24.6)$

For the wrench

1- **strength** $= \left|\vec{R}\right| = \sqrt{928^2 + 871^2 + 621^2}$= 1416 N

2- **parallel moment** = $M_{\parallel} = \dfrac{\vec{R}\ .\ \overrightarrow{M_A}}{\left|\vec{R}\right|}$

$$= \frac{(928,871,621)\ .\ (49.4,-90.5,-24.6)}{1416} = -34.1\ \text{N.m}$$

$$\overrightarrow{M_{\parallel}} = M_{\parallel} * \frac{\vec{R}}{\left|\vec{R}\right|} = -34.1 * \frac{(928,871,621)}{1416} = (-22.3,-20.9,-14.9)$$

N.m

Orthogonal moment = $\overrightarrow{M_{\perp}} = \overrightarrow{M_R} - \overrightarrow{M_{\parallel}}$

=$(49.4, -90.5, -24.6)$ - (-22.3,-20.9,-14.9) =(71.7,-69.6, -9.7) N.m

3- **Pitch** = P = $\dfrac{M_{\parallel}}{\left|\vec{R}\right|} = \dfrac{34.1}{1416}$= 0.024 m

- 4- **Central axis=** $\vec{r} = \frac{\vec{R} \times \overrightarrow{M_A}}{|\vec{R}|^2} + \lambda \vec{R}$ (λ constant parameter)

$$= \frac{(928,871,621) x (49.4,-90.5,-24.6)}{1416^2} + \lambda(928,871,621)$$

=(0.017, 0.027,-0.063) + λ (928,871,621)

- ***The intersection with the x-y plane***
- The point at the plane x-y = (x1,y1,0)
- x1= 0.017 +928 λ (1)
- y1= 0.027 +871 λ (2)
- 0= -0.063+621 λ (3)
- From (3) λ =0.063/621=1/10000
- From (1) x1= 0.11 & From (2) y1 =0.114
- The point of intersection =(0.11, 0.114,0)

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